



Overview course module “Stochastic Modelling”

I. Introduction

II. Actor-based models for network evolution

III. Co-evolution models for networks and behaviour

A. Extension of modelling framework to “behavior”

B. Homogeneity bias / network autocorrelation

C. An example: Music taste, alcohol & friendship

IV. Exponential **R**andom **G**raph **M**odels



Interdependence of networks and behavior

Social network dynamics often depend on actors' characteristics...

– **patterns of homophily:**

- interaction with similar others can be more rewarding than interaction with dissimilar others

– **patterns of exchange:**

- selection of partners such that they complement own abilities



...vice versa, also actors' characteristics can depend on the social network:

– **patterns of assimilation:**

- spread of innovations in a professional community
- pupils copying 'chic' behaviour of friends at school
- traders on a market copying (allegedly) successful behaviour of competitors

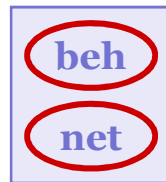
– **patterns of differentiation:**

- division of tasks in a work team



A. Extension of the network modelling framework

- Stochastic process in the (extended!) space of all possible network-behaviour configurations



- Again, the first observation is not modelled but conditioned upon as the process' starting value.
- Change is modelled as occurring in continuous time, but now there are two types of change.



Actor based approach now in two domains

- Network actors drive the process: individual decisions.
 - two domains of decisions:
 - decisions about **network** neighbours,
 - decisions about own **behaviour**.
 - per decision domain two submodels:
 - *When* can actor **i** make a decision? (**rate** functions)
 - *Which* decision does actor **i** make? (**objective** functions)



Micro steps that are modelled explicitly

Let $(\mathbf{x}, \mathbf{z})(\mathbf{t})$ be the state of the co-evolution process at time point \mathbf{t} (where \mathbf{x} stands for the network part and \mathbf{z} for the behaviour vector).

network micro steps

$(\mathbf{x}, \mathbf{z})(\mathbf{t}_1)$ and $(\mathbf{x}, \mathbf{z})(\mathbf{t}_2)$ differ in one tie variable \mathbf{x}_{ij} only.

behavioural micro steps

$(\mathbf{x}, \mathbf{z})(\mathbf{t}_1)$ and $(\mathbf{x}, \mathbf{z})(\mathbf{t}_2)$ differ (by one) in one behavioural score variable \mathbf{z}_i only.



Schematic overview of model components

	Timing of decisions	Decision rules
Network evolution	Network rate function	Network objective function
Behavioural evolution	Behaviour rate function	Behaviour objective function

- › *By simultaneously operating both processes on the same state space (conditionally independent, given the current state), feedback processes are instantiated.*
- › *Network evolution model and behavioural evolution model therefore are controlling for each other!*



Behavioural micro step by an actor

Choice options:

(1) increase, (2) decrease, or (3) keep score
on the behavioural variable

Choice probabilities:

Analogous to network part: multinomial logit model.

Effects on behaviour:

Also analogous to network part: based on statistics.



*Also here,
 many effects
 are possible
 to include in
 the objective
 function...*

TABLE 3
 SELECTION OF POSSIBLE EFFECTS FOR MODELING BEHAVIORAL EVOLUTION

effect	network statistic	effective transitions in network*	verbal description
1. tendency	z_i		main behavioral tendency
2. indegree × behavior	$z_i \sum_j x_{ji}$		effect of own popularity on behavior
3. outdegree × behavior	$z_i \sum_j x_{ij}$		effect of own activity on behavior
4. dense triads × behavior	$z_i \sum_{j,h} \text{group}(ijh)$		effect of belonging to cohesive subgroups on behavior
5. peripheral × behavior	$z_i \sum_{j,h,k} \text{peripheral}(i;jhk)$		effect of being peripheral to cohesive subgroups on behavior
6. isolation × behavior	$z_i \text{isolate}(i)$		effect of being isolated in the network on behavior
7. similarity	$\sum_j x_{ij} \text{sim}_{ij}$		assimilation to friends (contagion / influence)
8. similarity × reciprocity	$\sum_j x_{ij} x_{ji} \text{sim}_{ij}$		assimilation to reciprocating friends
9. similarity × pop. alter	$\sum_j x_{ij} \text{sim}_{ij} \sum_k x_{kj}$		assimilation to popular friends
10. similarity × dense triads	$\sum_{j,h} \text{group}(ijh)(\text{sim}_{ij} + \text{sim}_{ih})$		assimilation to the majority behavior in a cohesive subgroup
11. similarity × peripheral	$\sum_{j,h,k} (\text{peripheral}(i;jhk) \times (\text{sim}_{ij} + \text{sim}_{ih} + \text{sim}_{ik}))$		assimilation to those cohesive subgroups one unilaterally attaches to

* In the *effective transitions* illustrations, it is assumed that the behavioral dependent variable is dichotomous and centered at zero; the color coding is = low score (negative), = high score (positive), = arbitrary score. Actor i is the actor who changes color z_i in the transition indicated by the double arrows. Illustrations are not exhaustive.



B. Explaining homogeneity bias / netw. autocorr.

Many networks show homogeneity bias (equivalently, there is network autocorrelation on the behavioural variable).

One measure (implemented in SIENA) is the statistic

$$\sum_j x_{ij} \text{sim}_{ij}$$

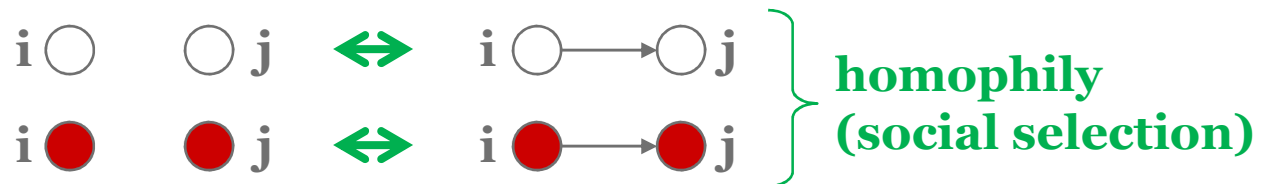
where sim_{ij} is a standardised measure of similarity of two actors on a variable z

$$\text{sim}_{ij} = 1 - \frac{|z_i - z_j|}{\text{range}_z}$$

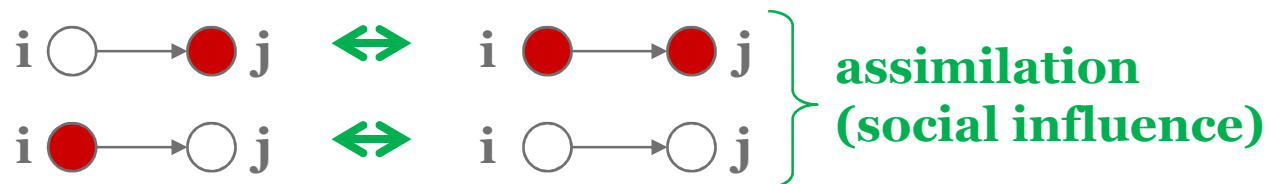


Competing explanatory stories

Actors base their social relations on similarity of individual features.



Actors adjust their individual features to the features of their social environment.





Modelling selection and influence

By including the statistic $\sum_j x_{ij} \text{sim}_{ij}$

...in the network objective function, homophilous selection is modelled,

...in the behaviour objective function, assimilation / social influence is modelled.

It can be of crucial importance to be able to control one effect for the occurrence of the other – e.g., in the design of social interventions to reduce smoking at school.



C: Example co-evolution analysis

A set of illustrative research questions:

1. *To what degree is music taste acquired via friendship ties?*
2. *Does music taste (co-)determine the selection of friends?*
3. *What is the role played by alcohol consumption in both friendship evolution and the dynamics of music taste?*

Data: *social network subsample of the **Scotland 11-16 Study***

(West & Sweeting, 1996)

three waves, 129 pupils (13-15 year old) at one school

pupils named up to 6 friends

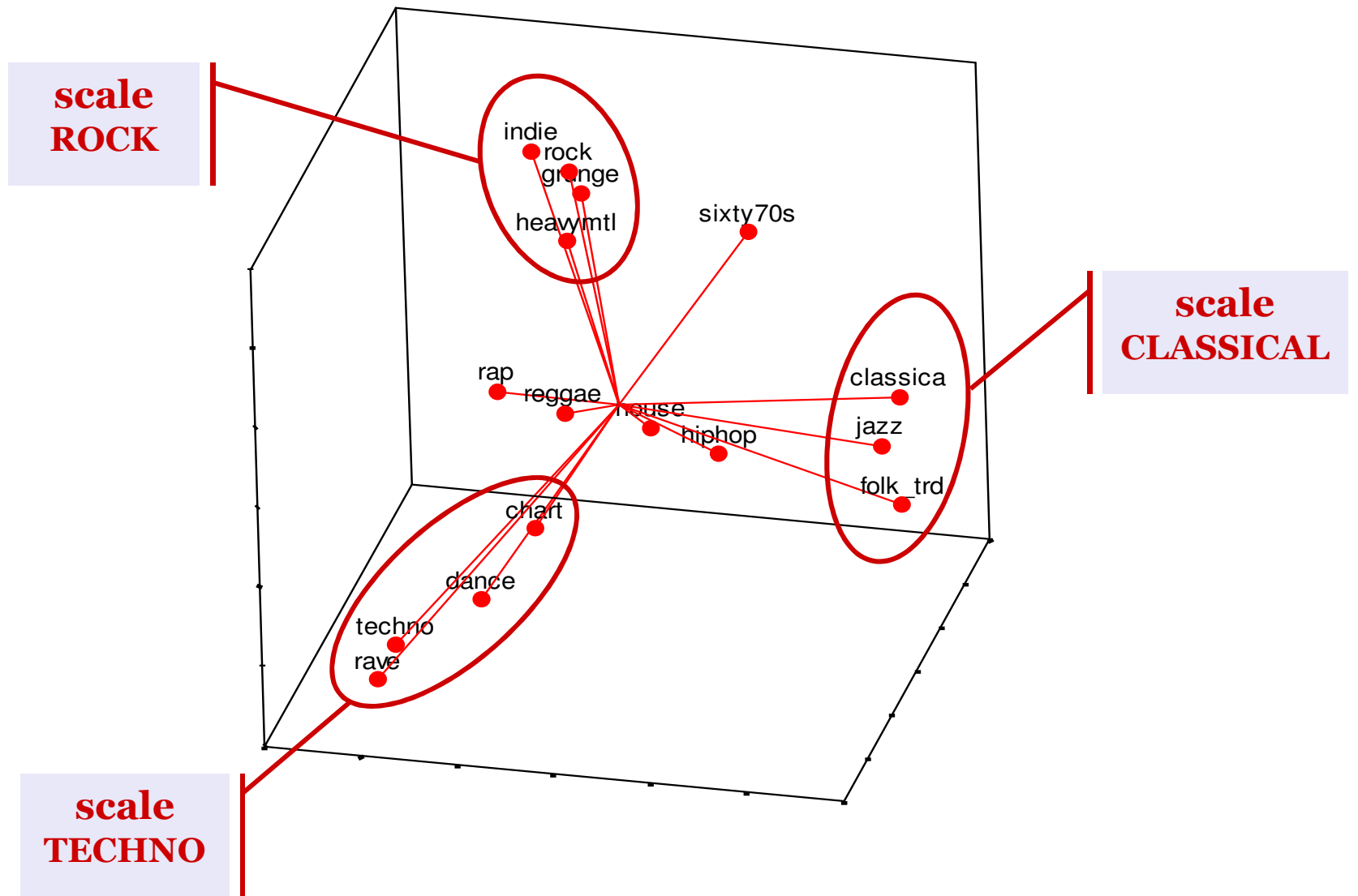


43. Which of the following types of music do you like listening to? Tick one or more boxes.

- | | | | |
|---------------------------|--------------------------|------------------|--------------------------|
| <i>Rock</i> | <input type="checkbox"/> | <i>Indie</i> | <input type="checkbox"/> |
| <i>Chart music</i> | <input type="checkbox"/> | <i>Jazz</i> | <input type="checkbox"/> |
| <i>Reggae</i> | <input type="checkbox"/> | <i>Classical</i> | <input type="checkbox"/> |
| <i>Dance</i> | <input type="checkbox"/> | <i>60's/70's</i> | <input type="checkbox"/> |
| <i>Heavy Metal</i> | <input type="checkbox"/> | <i>House</i> | <input type="checkbox"/> |
| <i>Techno</i> | <input type="checkbox"/> | <i>Grunge</i> | <input type="checkbox"/> |
| <i>Folk/Tradit.</i> | <input type="checkbox"/> | <i>Rap</i> | <input type="checkbox"/> |
| <i>Rave</i> | <input type="checkbox"/> | <i>Hip Hop</i> | <input type="checkbox"/> |
| <i>Other (what?).....</i> | | | |

Before applying SIENA: data reduction to informative dimensions...

Principal components analysis (confirmed by Mokken scaling) yields three music listening dimensions...





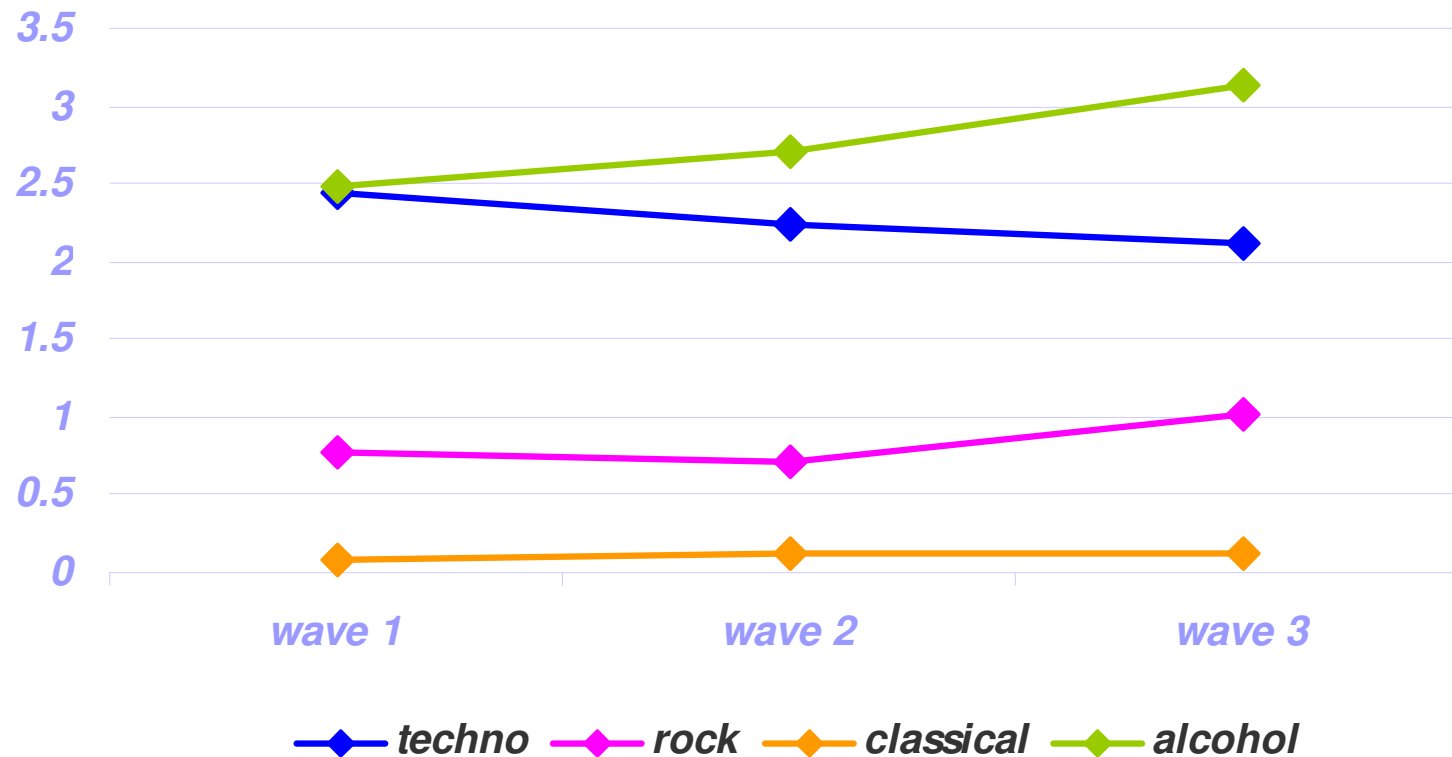
Alcohol question: five point scale

32. How often do you drink alcohol? Tick one box only.

- | | | |
|--------------------------------|--------------------------|----------|
| <i>More than once a week</i> | <input type="checkbox"/> | 5 |
| <i>About once a week</i> | <input type="checkbox"/> | 4 |
| <i>About once a month</i> | <input type="checkbox"/> | 3 |
| <i>Once or twice a year</i> | <input type="checkbox"/> | 2 |
| <i>I don't drink (alcohol)</i> | <input type="checkbox"/> | 1 |

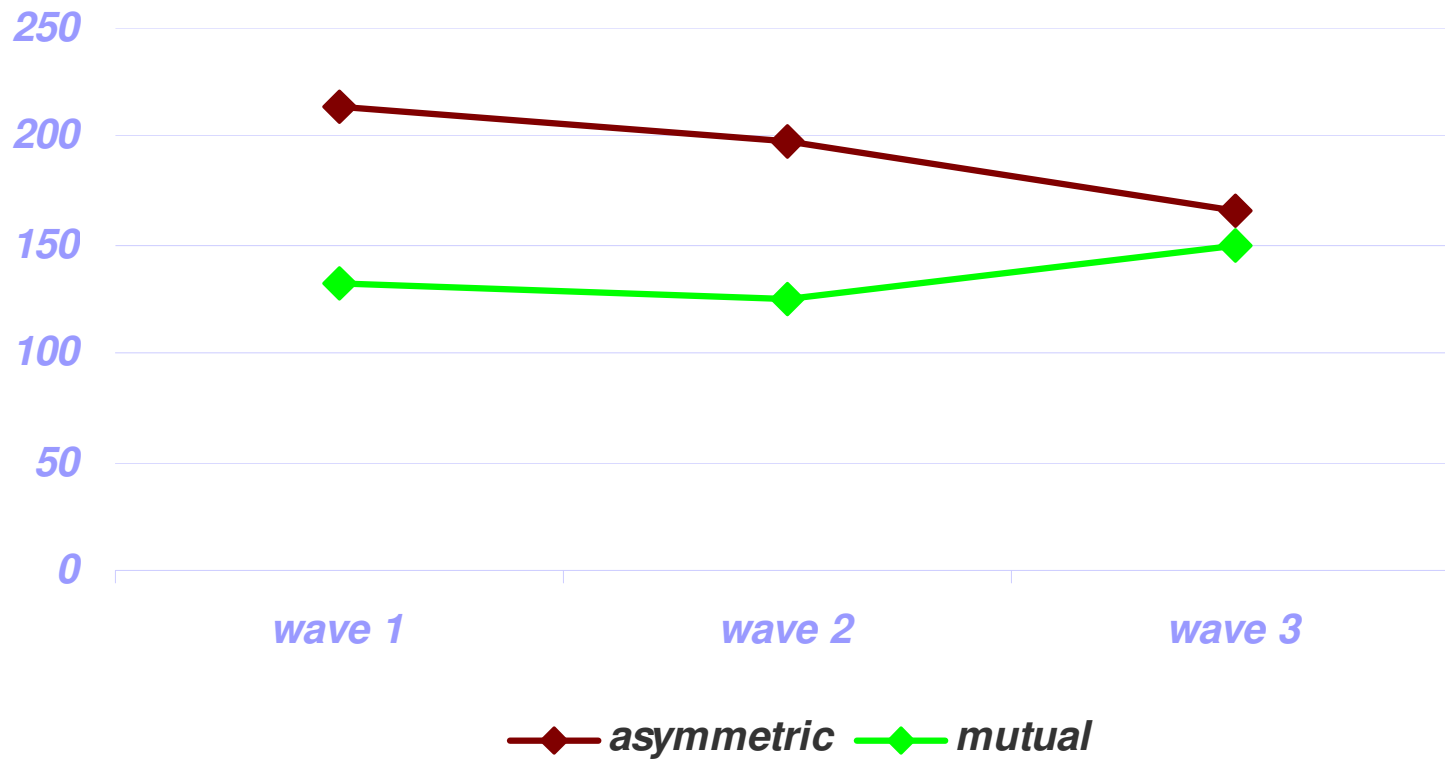


Average dynamics of the four behavioural variables...





...and global dynamics of friendship (dyad counts)





Analysis of the music taste data

Network objective function:

- intercept:
outdegree
- covariate-determined:
gender homophily
gender ego
gender alter
- network-endogenous:
reciprocity
distance-2
- behaviour-determined:
beh. homophily
beh. ego
beh. alter

Rate functions were kept as simple as possible (periodwise constant).

“behaviour” stands shorthand for the three music taste dimensions and alcohol consumption.



Behaviour objective function(s):

- intercept:
tendency
- network-determined:
assimilation to neighbours
- covariate-determined:
gender main effect
- behaviour-determined:
behaviour main effect



Results: network evolution

		parameter	s.e.	t-score
outdegree		-1.89	0.29	-6.51
reciprocity		2.34	0.12	20.08
distance-2		-1.09	0.07	-14.89
gender	sim	0.80	0.12	6.72
	alter	-0.21	0.12	-1.73
	ego	0.24	0.11	2.17
techno	sim	0.08	0.33	0.26
	alter	0.07	0.05	1.30
	ego	-0.10	0.05	-1.93
rock	sim	0.11	0.41	0.26
	alter	0.19	0.07	2.75
	ego	-0.07	0.08	-0.92
classical	sim	1.44	0.69	2.07
	alter	0.15	0.17	0.91
	ego	0.40	0.17	2.42
alcohol	sim	0.83	0.27	3.08
	alter	-0.03	0.04	-0.75
	ego	-0.03	0.03	-0.85

Low overall density in these networks.

Reciprocation is important for friendship.

There is a tendency towards transitive closure.



Results: network evolution

		parameter	s.e.	t-score
outdegree		-1.89	0.29	-6.51
reciprocity		2.34	0.12	20.08
distance-2		-1.09	0.07	-14.89
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	alter	-0.21	0.12	-1.73
	ego	0.24	0.11	2.17
techno	sim	0.08	0.33	0.26
	alter	0.07	0.05	1.30
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	alter	-0.03	0.04	-0.75
	ego	-0.03	0.03	-0.85

There is gender homophily:

		alter	
		boy	girl
ego	boy	0.38	-0.62
	girl	-0.18	0.41

table gives gender-related contributions to the objective function

There is alcohol homophily:

		alter	
		low	high
ego	low	0.36	-0.59
	high	-0.59	0.13

table shows contributions to the objective function for highest / lowest possible scores



Results: network evolution

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outdegree		-1.89	0.29	-6.51
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Techno style listeners are marginally less active in sending friendship nominations.

Rock style listeners are more popular as friends.

Classical style listeners select each other as friends!

Classical style listeners are more active in sending friendship nominations.



Results: behavioural evolution

	alcohol		techno		rock		classical	
	par.	s.e.	par.	s.e.	par.	s.e.	par.	s.e.
intercept	-0.30	0.37	0.01	0.25	0.59	0.25	0.67	1.30
assimilation	0.94	0.27	0.45	0.18	0.63	0.28	0.42	1.17
gender	-0.06	0.19	0.25	0.12	0.01	0.19	1.57	0.83
techno	0.23	0.16	---	---	-0.25	0.09	-0.46	0.40
rock	0.16	0.16	-0.34	0.10	---	---	0.64	0.39
classical	-0.59	0.32	-0.13	0.23	-0.34	0.30	---	---
alcohol	---	---	0.07	0.10	-0.11	0.07	-1.03	0.34

- Assimilation to friends occurs:
 - on the alcohol dimension,
 - on the techno dimension,
 - on the rock dimension.



Results: behavioural evolution

	alcohol		techno		rock		classical	
	par.	s.e.	par.	s.e.	par.	s.e.	par.	s.e.
intercept	-0.30	0.37	0.01	0.25	0.59	0.25	0.67	1.30
assimilation	0.94	0.27	0.45	0.18	0.63	0.28	0.42	1.17
gender	-0.06	0.19	0.25	0.12	0.01	0.19	1.57	0.83
techno	0.23	0.16	---	---	-0.25	0.09	-0.46	0.40
rock	0.16	0.16	-0.34	0.10	---	---	0.64	0.39
classical	-0.59	0.32	-0.13	0.23	-0.34	0.30	---	---
alcohol	---	---	0.07	0.10	-0.11	0.07	-1.03	0.34

- There is evidence for mutual exclusiveness of:
 - listening to techno and listening to rock,
 - listening to classical and drinking alcohol.
- The classical listeners tend to be girls.



Conclusions from the analysis

Does music taste (co-)determine the selection of friends?

Somewhat.

- There is no universal music taste homophily (possible exception: classical style).
- Listening to rock music seems to coincide with popularity,
- listening to classical music with unpopularity.

To what degree is music taste acquired via friendship ties?

It depends on the specific music taste:

- Listening to techno or rock music is ‘learnt’ from peers,
- listening to classical music is not – maybe a ‘parental home thing’?



What is the role played by alcohol consumption in friendship formation?

There is homophilous selection going on:

- Friends select each other based on similarity in alcohol consumption.

What is the role played by alcohol consumption in the dynamics of music taste?

Only for the classical scale, an effect was found:

- Drinking alcohol reduces the chances of listening to classical music (and vice versa).