



## Overview course module “Stochastic Modelling”

### I. Introduction

### II. Actor-based models for network evolution

- A. Data requirements
- B. Modelling principles & assumptions
- C. The network evolution algorithm
- D. Model components
- E. An example

### III. Co-evolution models for networks and actor properties

### IV. Exponential Random Graph Models



## A. Continuous-time modelling, discrete-time data

Required are repeated measures of the same network:

- same group of actors  
(some composition change is allowed)
- same relational variable. **\*states, not events!\***

Subsequent measures are assumed to be related through a continuous process of change.

*In principle, continuous-time data should be easier to analyse this way – but the methods are not (yet) implemented.*



## Example data: (Andrea Knecht, 2003/04)

Networks among first grade pupils at Dutch secondary schools (“bridge class”).

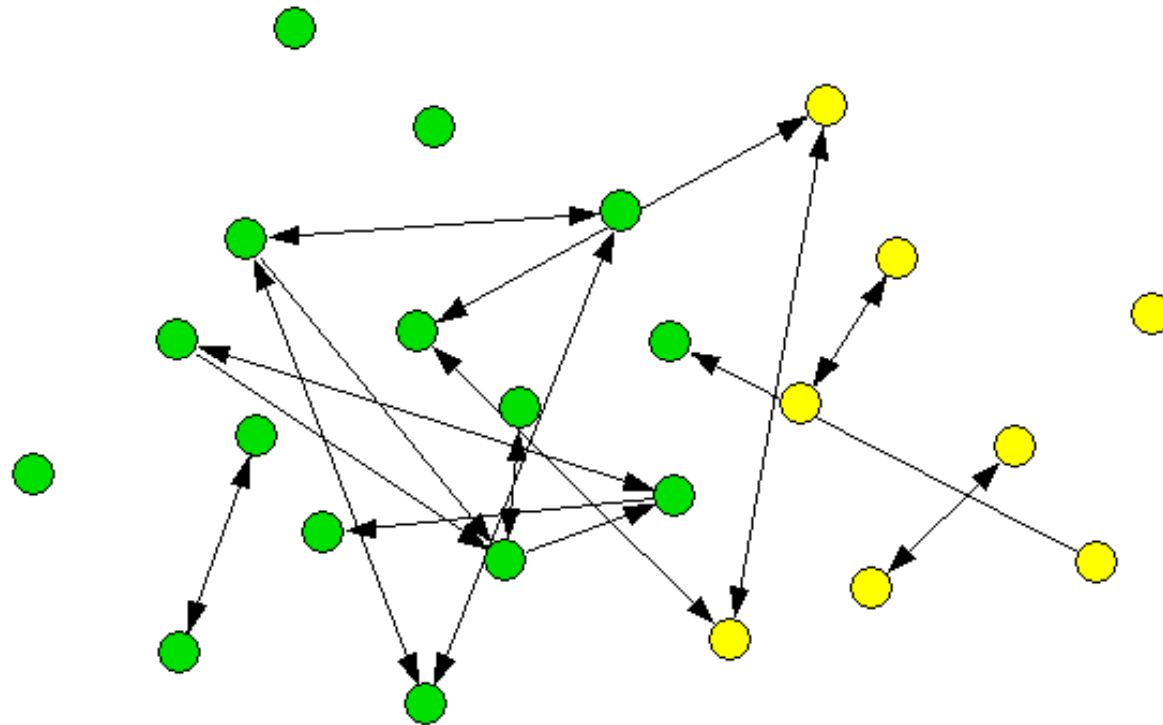
125 school classes

4 measurement points,

various network & individual measures.

The following slides show the evolution of the friendship network in one classroom.

*The graph layout is a bit messy for each observation alone, but optimal over time according to the Kamada-Kawai algorithm.*



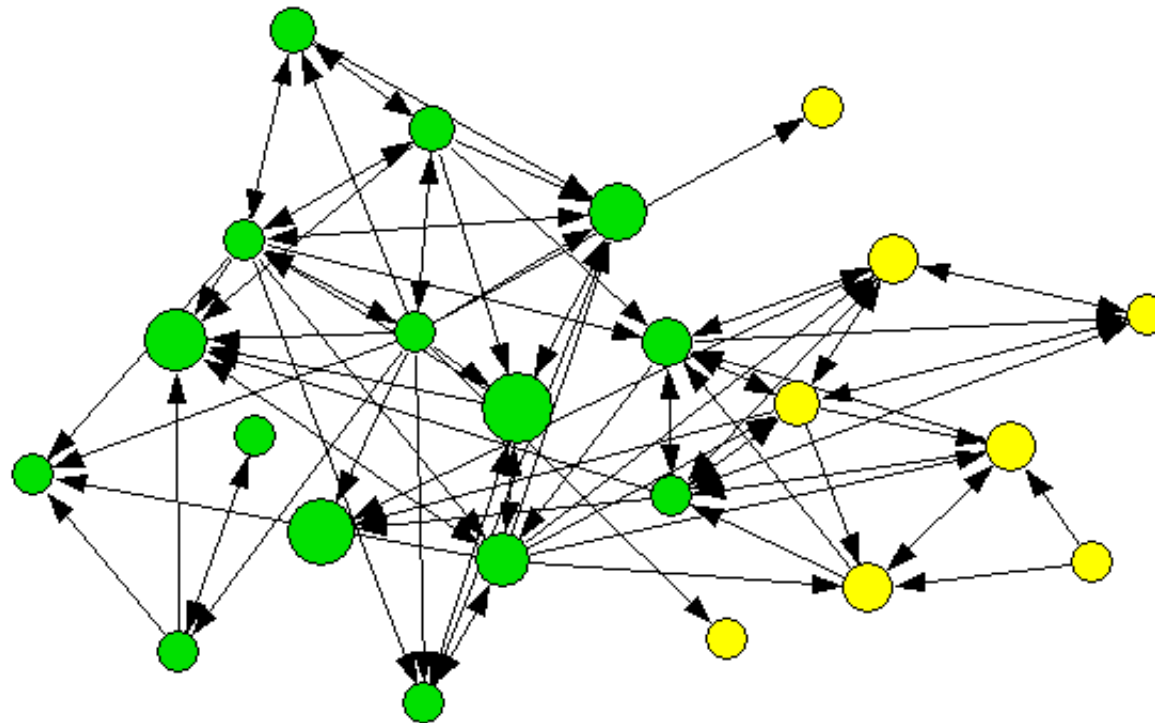
Friendship ties inherited from primary school  
 girls yellow boys green



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1st wave: August/September 2003

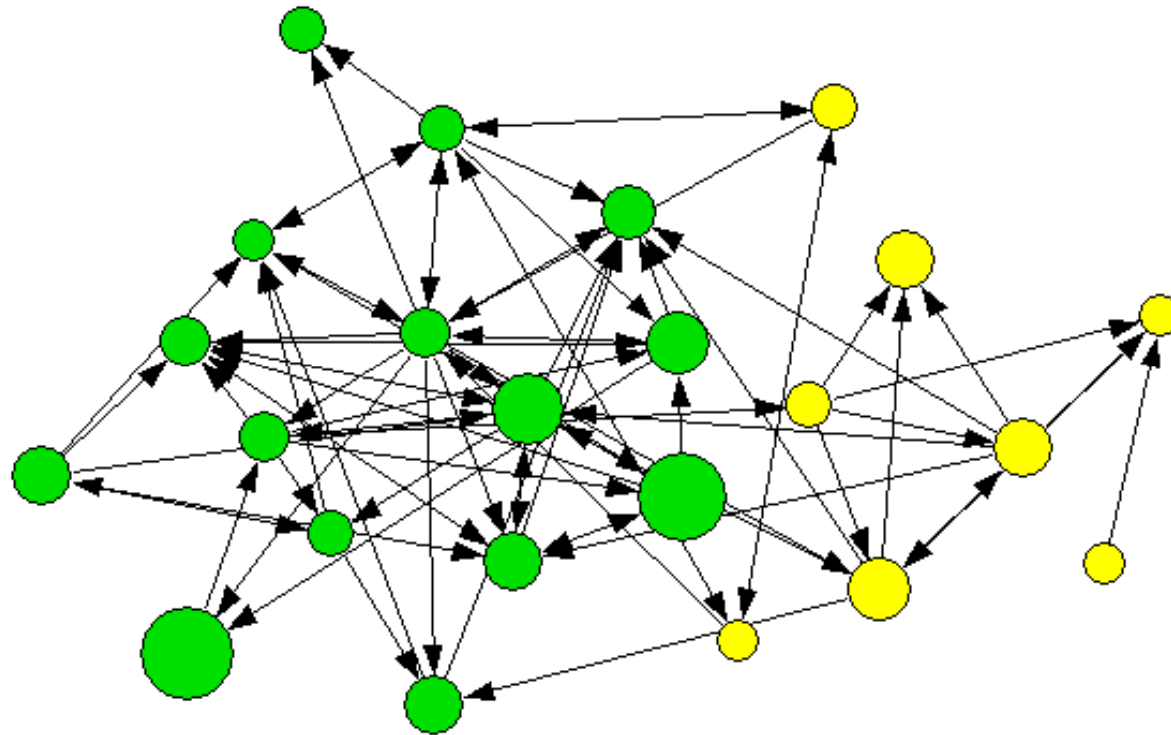
node size indicates strength of delinquency



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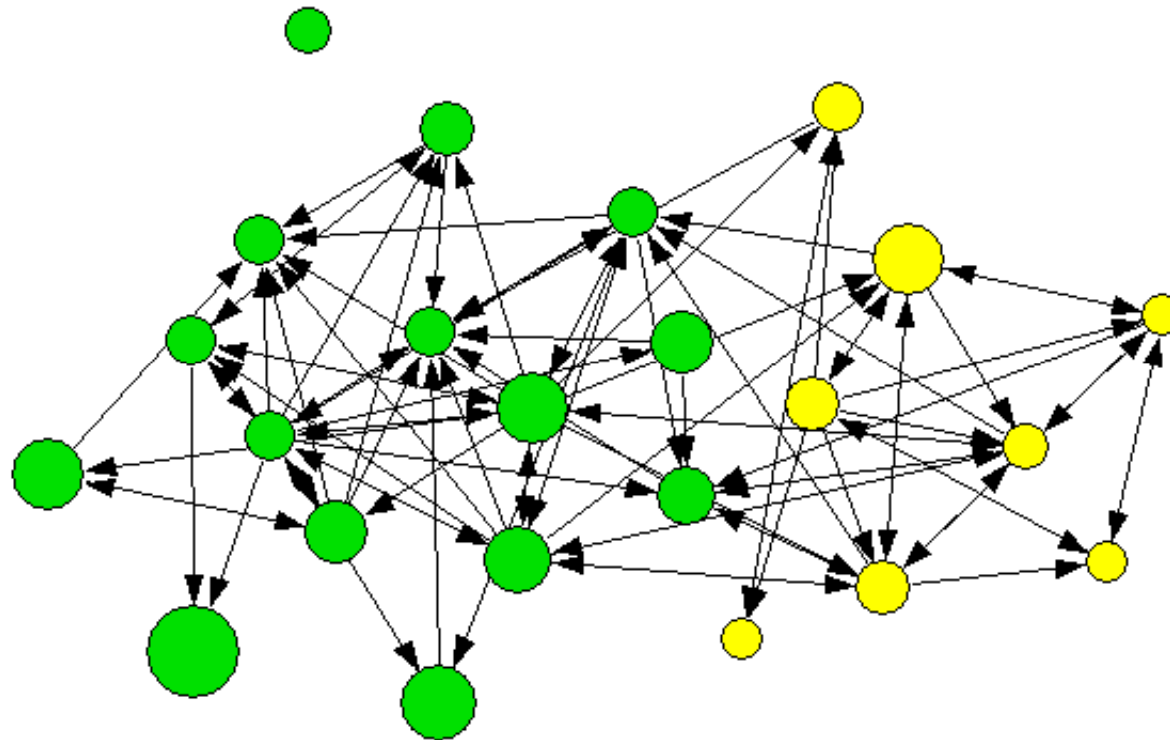
2nd wave: November/December 2003



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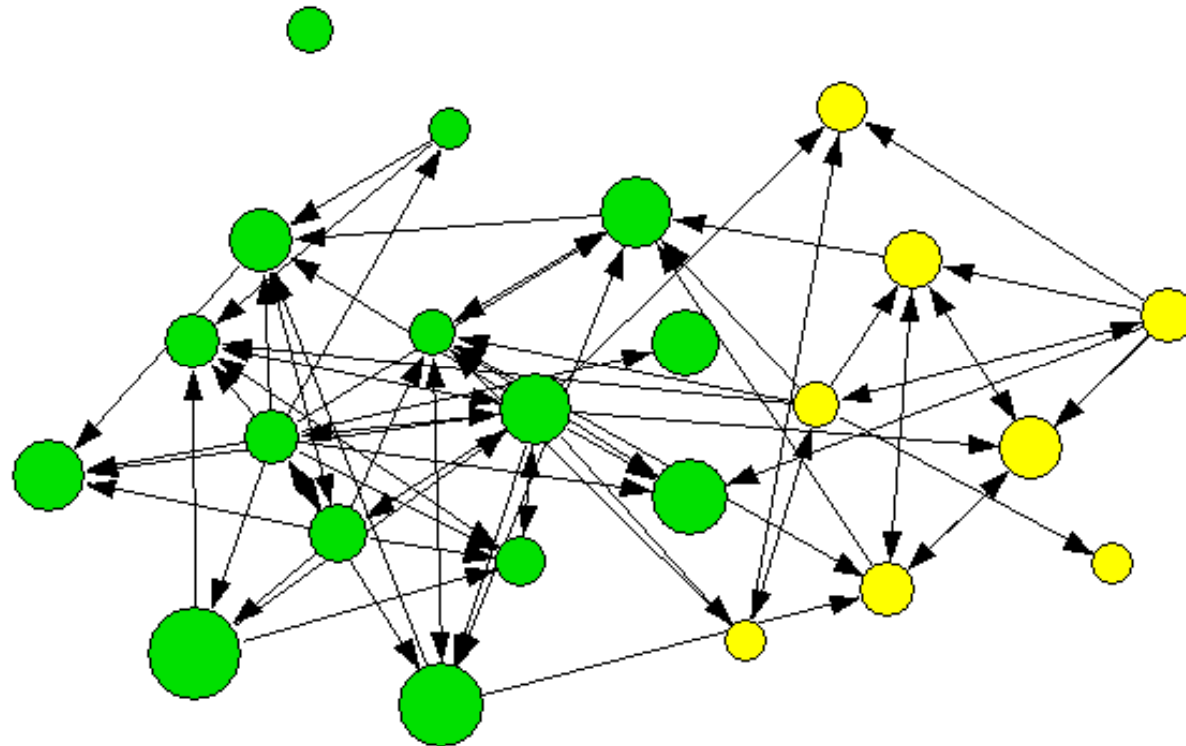
3rd wave: February/March 2004



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4th wave: May/June 2004



## *Points to consider before trying actor-based modelling*

### **states** ↔ **events**

NOT snapshots of e-mail traffic, BUT reliable measures of a social relation.

*Event networks could be aggregated over time to obtain state networks!*

### **change** ↔ **stability**

The networks should change ‘slowly’, contain a stable part.

*Rules for structural change typically are about individual ties changing in response to surrounding ties (which remain stable, for that moment).*



## *Data format issues to consider*

**binary** ↔ **signed** ↔ **valued**

**directed** ↔ **undirected**

**tie loss possible** ↔ **growth only networks**

**1-mode** ↔ **bipartite**

**single dependent** ↔ **multiplex**

The standard model is developed for a single dependent, binary, directed, 1-mode network that can both grow and shrink over time.

*Everything else is a non-standard model extension, and not necessarily supported by the software implementation.*



## B. Modelling principles for such data sets

**Random walk:** Network evolution proceeds as a stochastic process on the space of all possible networks;

**No contamination by the past:** The first observation is not modelled but conditioned upon as the process' starting value.

**Continuous-time model:** Change is modelled as occurring in continuous time.

**Micro steps:** Big change from one observation to the next is assumed to accrue from a sequence of smallest changes.



## ***One more modelling principle***

***Actor-driven model:*** network actors are the locus of modelling, change is due to individual decisions\*.

- actors control “their” network ties;
- two submodels:
  - When can actor  $i$  make a decision? (**rate** function)
  - Which decision does actor  $i$  make? (**objective** function)

***Technically: Continuous time Markov process.***

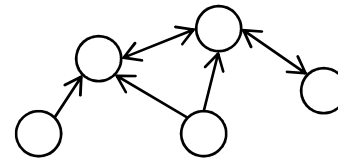
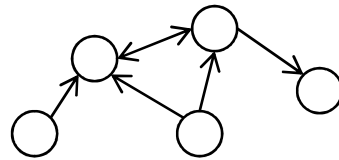
\* assumption: Luce’s (1959) choice axioms; decisions are assumed to be *conditionally independent of each other, given the current state.*



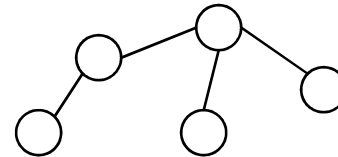
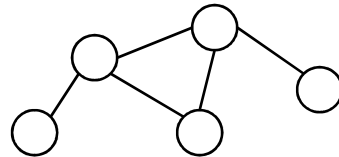
## Neighbouring states in the network space...

- › ... are networks that differ by just one tie variable, all others are identical.

- Example directed network:



- Example undirected network:



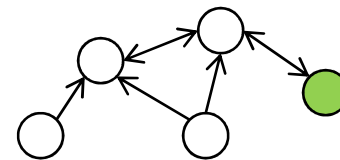
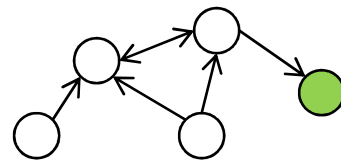
Terminology: these networks differ by a ‘micro step’



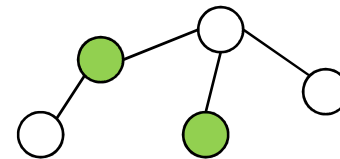
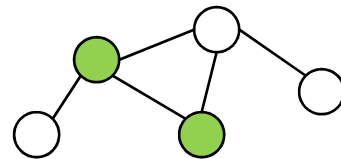
## Micro steps...

› ... involve uniquely identified actors – these are assumed to control the tie variable:

- directed network: ONE actor

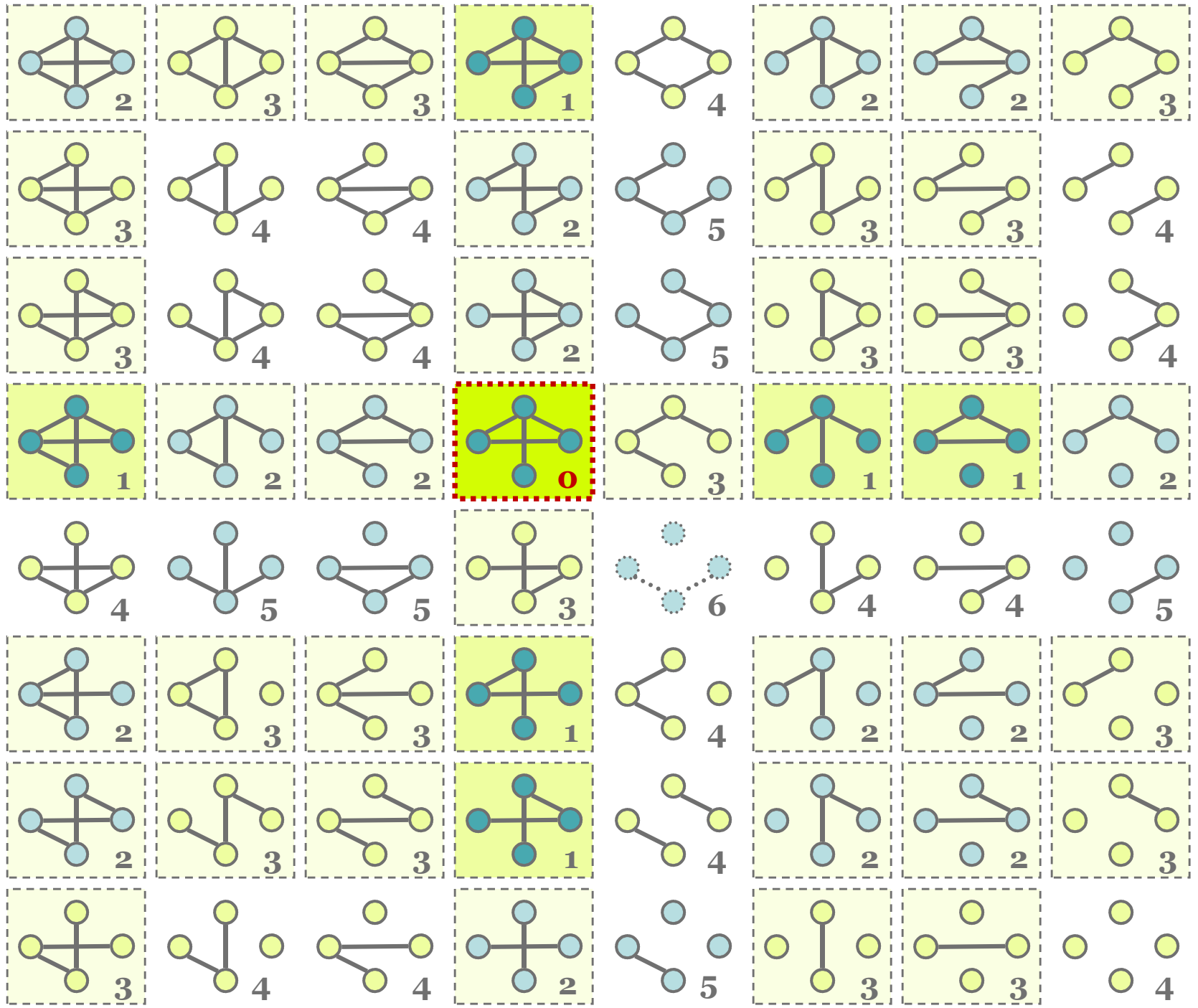


- undirected network: TWO actors



The directed case is therefore simpler to model, in an actor-based way.

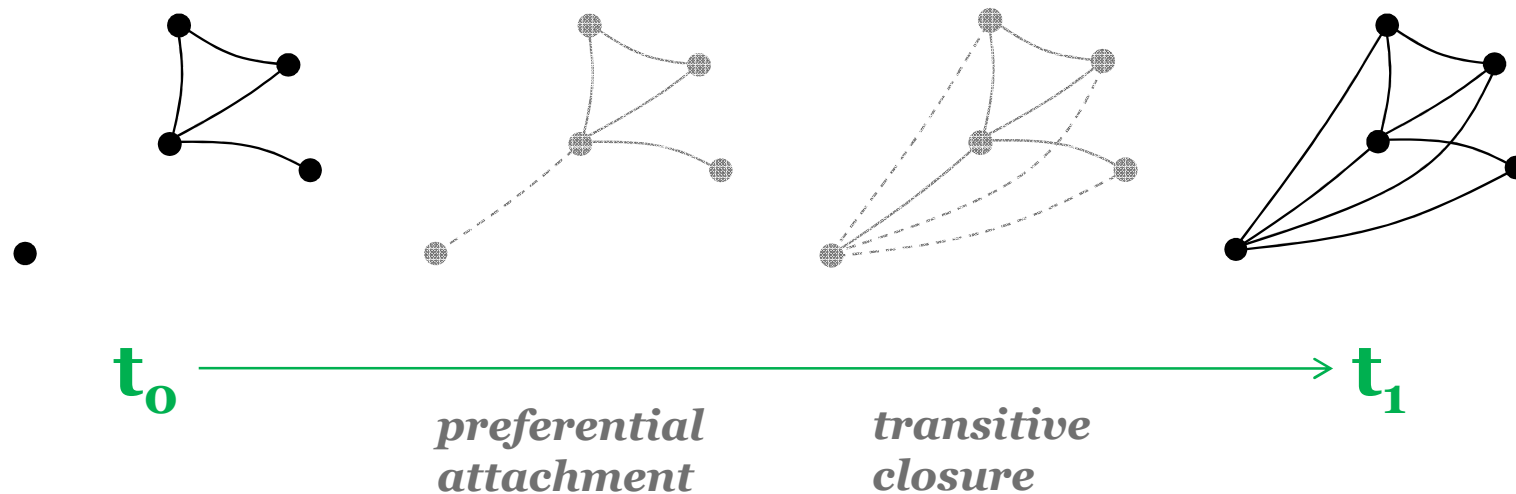
Example: Distances from **0**-network in 'micro steps'





## *Nice feature of continuous-time modelling*

*Complex patterns emerging from simple(r) mechanisms*



Some new ties may be realisation-contingent on other new ties.  
Discrete time models cannot easily model their compound emergence.



## C: The network evolution algorithm

*Network evolution in observation period  $t_0 \rightarrow t_1$  takes place as follows:*

1. Model time is set to  $t = t_0$ , and simulation starts out at the network observed at this time point.
2. For all actors, a waiting time is sampled according to the *rate function*.
3. The actor with the shortest waiting time  $\tau$  is identified.
4. If  $t + \tau > t_1$ , the simulation terminates.
5. Otherwise, the identified actor gets the opportunity to set a micro step. This is determined by his *objective function*.
6. Simulation proceeds with step 2.

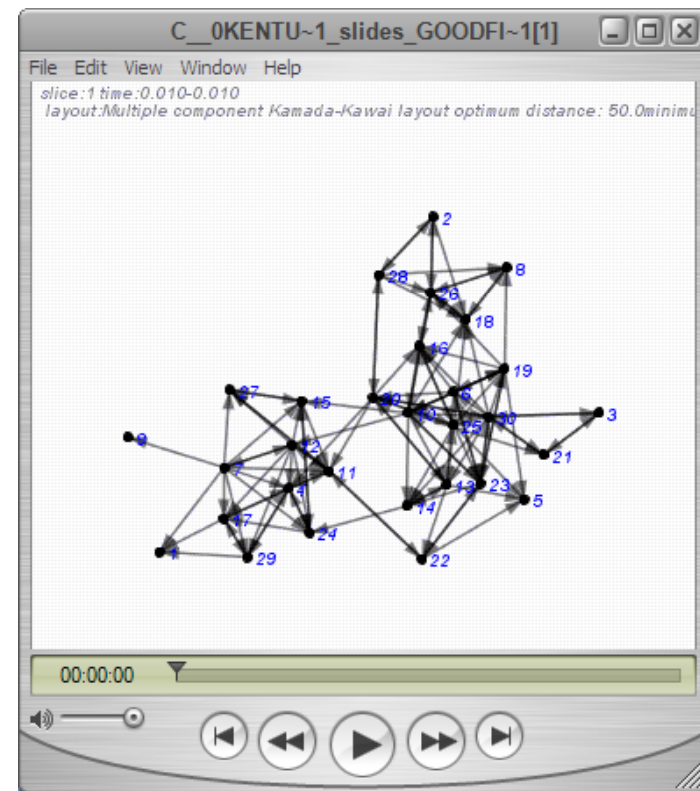


## *Visualisation with SONIA*

SIENA-based imputation of the unobserved trajectory of changes between two consecutive observations.

The movie shows but *one* instantiation of the model.

*Classroom friendship data,  
 Andrea Knecht, 2003/04.*





## *The rate function*

$$\lambda_i(\mathbf{x}) = \sum_k \rho_k r_{ik}(\mathbf{x})$$

- › Models *speed* differences between actors  $\mathbf{i}$ .
- › Statistics  $r_{ik}$  of  $\mathbf{i}$ 's neighbourhood in  $\mathbf{x}$  are weighted by model parameters  $\rho_k$ .
- › These weights express whether the feature expressed in the statistic is related to more frequent ( $\rho_k > 0$ ) or less frequent ( $\rho_k < 0$ ) network changes by the actors.
- › They are estimated from the data.

***Technically,  $\lambda_i$  is parameter of an exponential distribution of waiting times.***

***Typically, it is good to start an analysis under the assumption of a periodwise constant rate function.***



## ***The objective function*** $f_i(\mathbf{x}) = \sum_k \beta_k s_{ik}(\mathbf{x})$

- › Models attractiveness of network states  $\mathbf{x}$  to actor  $\mathbf{i}$ .
- › Statistics  $s_{ik}$  of  $\mathbf{i}$ 's neighbourhood in  $\mathbf{x}$  are weighted by model parameters  $\beta_k$ .
- › These weights express whether the feature expressed in the statistic is desired ( $\beta_k > 0$ ) or averted ( $\beta_k < 0$ ).
- › Also they are estimated from the data.

***Technically,  $f_i(\mathbf{x})$  is parameter of a multinomial logit model for discrete, probabilistic choice.***

***The objective function is the main part of modelling. Here, hypotheses typically are operationalised.***

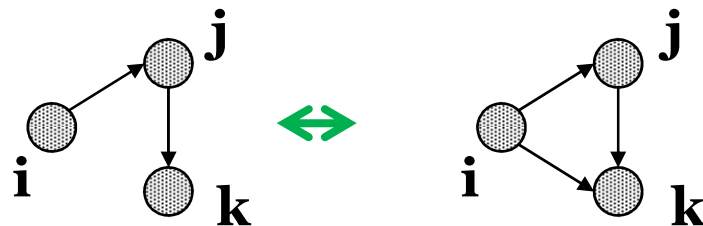


## Some effect statistics

*reciprocity effect*  $S_{i \text{ recip.}} = \sum_j X_{ij} X_{ji}$



*transitivity effect*  $S_{i \text{ tr.trip.}} = \sum_{jh} X_{ij} X_{jh} X_{jh}$



*Very often, effect statistics are motif (subgraph) counts.*

Effects measure attractiveness difference between right and left configuration, for the focal actor **i**.

**Many other effects are possible to include in the objective function...**

TABLE 2  
SELECTION OF POSSIBLE EFFECTS FOR MODELING NETWORK EVOLUTION

effect	network statistic	effective transitions in network*	verbal description
1. outdegree	$x_{ij}$		preference for ties to arbitrary others
2. reciprocity	$x_{ij}x_{ji}$		preference for reciprocated ties
3. transitive triplets	$x_{ij} \sum_h x_{ih} x_{hj}$		preference for being friend of the friends' friends
4. balance	$x_{ij} \text{strsim}_{ij}$		preference for ties to structurally similar others
5. actors at distance two	$\begin{cases} 1 & \text{if between}(h,ij) = 1 \text{ for some } h \\ 0 & \text{else} \end{cases}$		preference for keeping others at social distance two
6. popularity alter	$x_{ij} \sum_h x_{jh}$		preference for attaching to popular others, i.e., others who are often named as friend ('preferential attachment')
7. activity alter	$x_{ij} \sum_h x_{jh}$		preference for attaching to active others, i.e., others who name many friends
8. 3-cycles	$x_{ij} \sum_h x_{jh} x_{hi}$		preference for forming relationship cycles (negative indicator for hierarchical relations)
9. betweenness	$\sum_h \text{between}(i,hj)$		preference for being in an intermediary position between unrelated others
10. dense triads	$\sum_h \text{group}(ijh)$		preference for being part of cohesive subgroups
11. peripheral	$\sum_{hk} \text{peripheral}(i,jhk)$		preference for unilaterally attaching to cohesive subgroups
12. similarity	$x_{ij} \text{sim}_{ij}$		preference for ties to similar others (selection)
13. behavior alter	$x_{ij} z_i$		main effect of alter's behavior on tie preference
14. behavior ego	$x_{ij} z_i$		main effect of ego's behavior on tie preference
15. similarity × reciprocity	$x_{ij} x_{ji} \text{sim}_{ij}$		preference for reciprocated ties to similar others
16. between dissimilar alters	$\sum_h (1 - \text{sim}_{jh}) \text{between}(i,jh)$		preference for being in an intermediary position between unrelated, dissimilar others (brokerage potential)
17. similarity × dense triads	$\sum_h \text{group}(ijh) (\text{sim}_{ij} + \text{sim}_{ih})$		preference for being part of behaviorally similar cohesive subgroups
18. behavior × peripheral	$z_i \sum_{hk} \text{peripheral}(i,jhk)$		behavior-specific preference for unilaterally attaching to cohesive subgroups
19. similarity × peripheral	$\sum_{hk} (\text{peripheral}(i,jhk) \times (\text{sim}_{ij} + \text{sim}_{ih} + \text{sim}_{jk}))$		preference for unilaterally attaching to behaviorally similar cohesive subgroups

\* In the *effective transitions* illustrations, it is assumed that the behavioral dependent variable is dichotomous and centered at zero; the color coding is  $\circ$  = low score (negative),  $\bullet$  = high score (positive),  $\ominus$  = arbitrary score. The tie  $x_{ij}$  from actor  $i$  to actor  $j$  is the one that changes in the transition indicated by the double arrow. Illustrations are not exhaustive.



## **Choice probabilities** $\Pr(x \rightarrow_i x') \propto \exp(f_i(x'))$

- › Choice probabilities for micro steps are proportional to the exponential function of the objective function.
- › Valid options are all possible micro steps, plus the option not to change the status quo.
- › This probability distribution can be interpreted as optimisation of a random utility function, namely the objective function  $f_i$  plus a Gumbel-distributed error term.
- › Note that the probabilities only depend on  $x'$  and not on past states, not even  $x$ . This can be relaxed (keyword: endowment function).



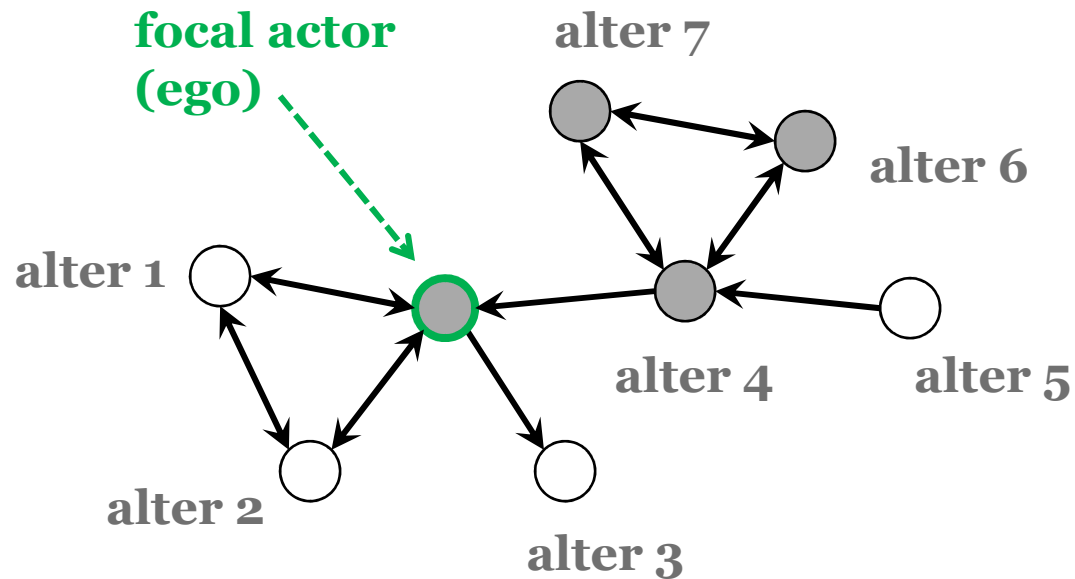
## *Example of a model specification*

Objective function parameter estimates (friendship):

- outdegree  $\beta_{\text{outdg.}} = -2.6$  *friendship is rare*
- reciprocity  $\beta_{\text{recip.}} = 1.8$  *friendship is reciprocal*
- transitivity  $\beta_{\text{tr.trip.}} = 0.4$  *friendship shows clustering*
- three-cycles  $\beta_{\text{3-cycl.}} = -0.7$  *friendship shows hierarchy*
- same gender  $\beta_{\text{same}} = 0.8$  *friendship is sex segregated*



## Example of an actor's decision

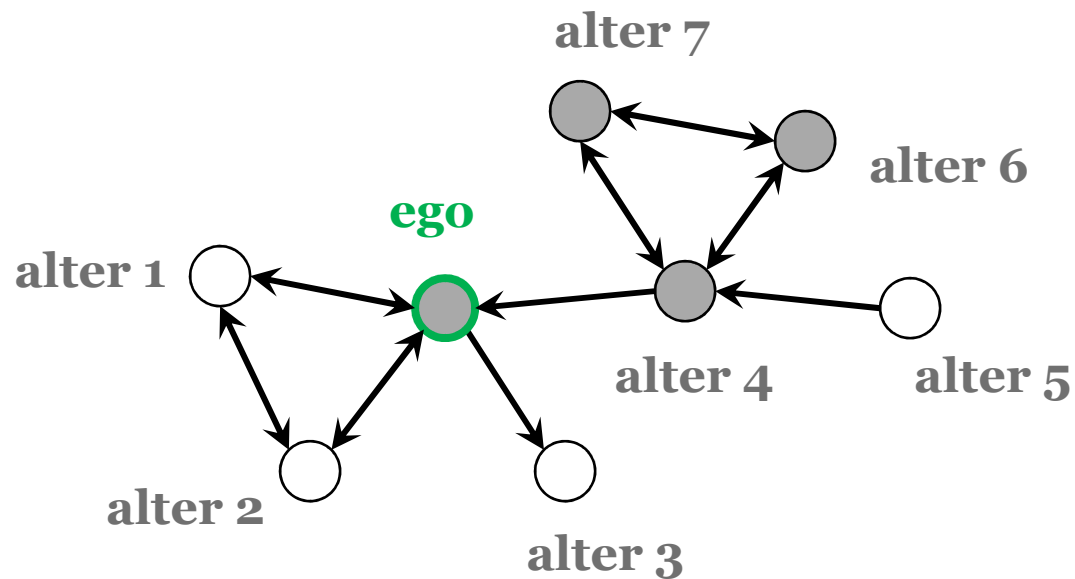


### Options:

- drop tie to alter 1
- drop tie to alter 2
- drop tie to alter 3
- create tie to alter 4
- create tie to alter 5
- create tie to alter 6
- create tie to alter 7
- keep status quo



## Count model-relevant motifs for all options

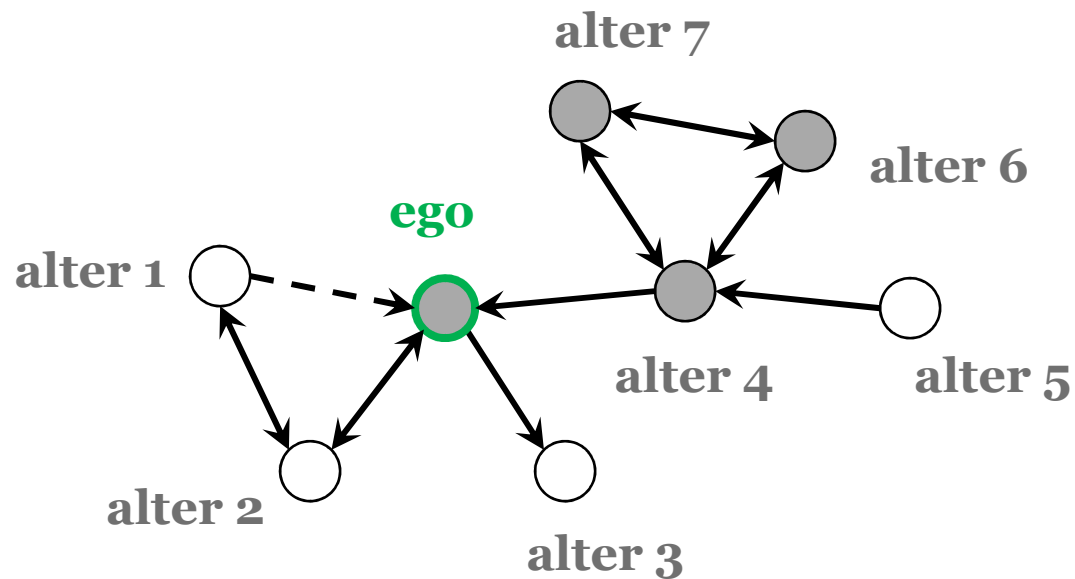


### *Status quo (ego):*

- 3 outgoing ties
- 2 reciprocated ties
- 2 transitive triplets
- 2 three-cycles
- 0 same colour ties



## Count model-relevant motifs for all options

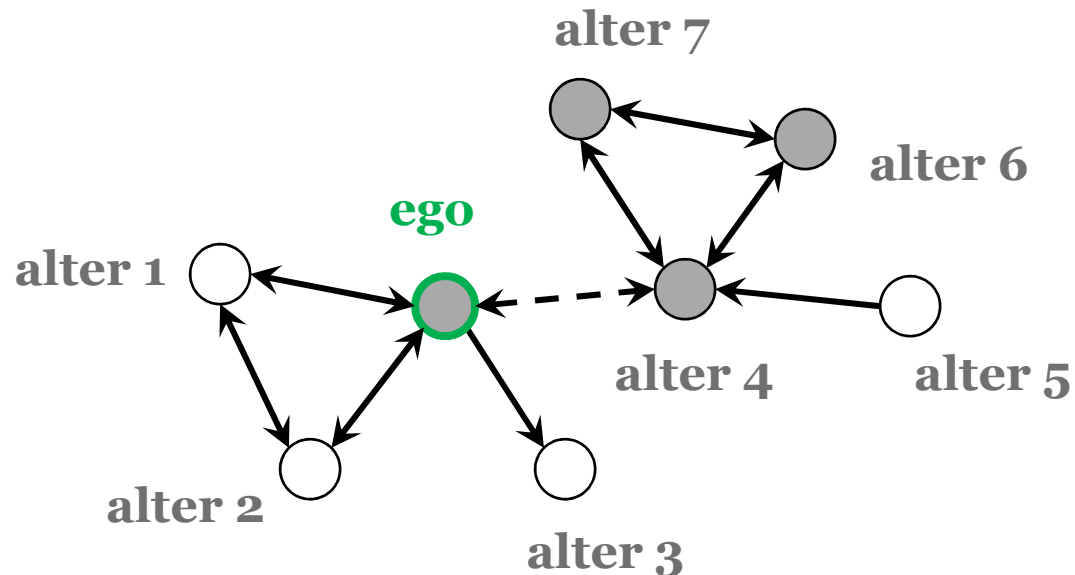


### Drop tie to alter 1:

- 2 outgoing ties
- 1 reciprocated tie
- 1 transitive triplet
- 1 three-cycle
- 0 same colour ties



## Count model-relevant motifs for all options



### Create tie to alter 4:

- 4 outgoing ties
- 3 reciprocated ties
- 2 transitive triplets
- 2 three-cycles
- 1 same colour tie

*...these calculations  
 are done for all the  
 eligible options.*



Option	# out-ties	# recip. ties	# tr.triplets	# 3-cycles	# same col.
drop tie to alter 1	2	1	1	1	0
drop tie to alter 2	2	1	1	1	0
drop tie to alter 3	2	2	2	2	0
create tie to alter 4	4	3	2	2	1
create tie to alter 5	4	2	2	3	0
create tie to alter 6	4	2	2	3	1
create tie to alter 7	4	2	2	3	1
keep status quo	3	2	2	2	0



## Calculation of objective function:

- $f_{\text{drop-1}}$
- $f_{\text{drop-2}}$
- $f_{\text{drop-3}}$
- $f_{\text{create-4}}$
- $f_{\text{create-5}}$
- $f_{\text{create-6}}$
- $f_{\text{create-7}}$
- $f_{\text{stat. quo}}$

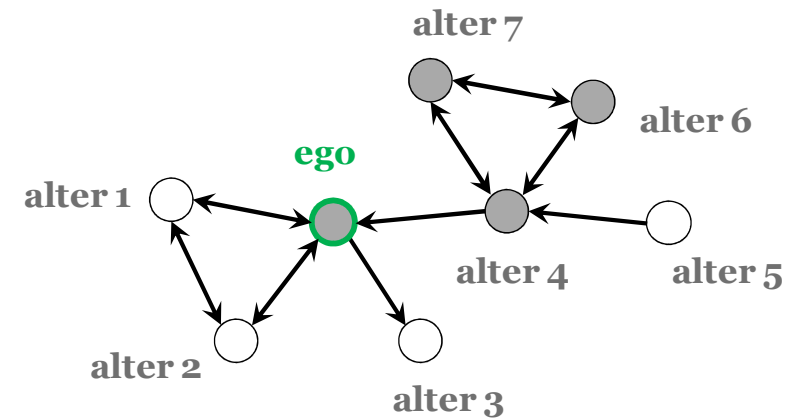
Option	# out-ties	# recip. ties	# tr.triplets	# 3-cycles	# same col.
drop tie to alter 1	2	1	1	1	0
drop tie to alter 2	2	1	1	1	0
drop tie to alter 3	2	2	2	2	0
create tie to alter 1	4	2	2	2	1
create tie to alter 2	4	2	2	3	0
create tie to alter 3	4	2	2	3	1
create tie to alter 4	4	2	2	3	1
keep status quo	3	2	2	2	0

**Matrix  $S_{\text{ego}}$**

- $\beta_{\text{outdg.}}$
- $\beta_{\text{recip.}}$
- $\beta_{\text{tr.trip.}}$
- $\beta_{\text{3-cycles}}$
- $\beta_{\text{same}}$



Option	objective function	exponential transform	probability
drop tie to alter 1	-3.7	0.025	14%
drop tie to alter 2	-3.7	0.025	14%
drop tie to alter 3	-2.2	0.111	62%
create tie to alter 4	-4.8	0.008	5%
create tie to alter 5	-8.1	0.000	0%
create tie to alter 6	-7.3	0.001	0%
create tie to alter 7	-7.3	0.001	0%
keep status quo	-4.8	0.008	5%



***Dropping the tie to alter 3 clearly dominates this decision situation.***

*Note: SIENA internally centers many variables – this does not affect the choice probabilities.*



## Local characterisation of choice probabilities

- For two networks that could be obtained in competing micro steps from the same network of origin, the ratio of choice probabilities is this (“odds”):

$$\frac{\Pr(x^c \rightarrow_i x^a)}{\Pr(x^c \rightarrow_i x^b)} = \exp \left( \sum_{k=1}^K \beta_k \left( s_{ik}(x^a) - s_{ik}(x^b) \right) \right)$$

compared are two moves (“micro steps”) made by actor  $i$  from a network  $x^c$  to two “neighbouring networks”  $x^a$  and  $x^b$

model parameters

difference in model statistics of actor  $i$  between the two compared moves



## *The main part of the formula in detail:*

The sum  $\sum_{k=1}^K \beta_k (s_{ik}(x^a) - s_{ik}(x^b))$  determines whether  $x^a$  or  $x^b$  is more likely to succeed  $x^c$  in the network evolution process.

$\beta_k$  positive: states with higher scores  $s_{ik}$  are more likely than states with lower scores;

$\beta_k$  negative: states with lower scores  $s_{ik}$  are more likely than states with higher scores.

This way, parameter values  $\beta_k$  express dynamic tendencies of network evolution: “*actors are moving towards a high [low] score on the corresponding network statistic  $s_{.k}$* ”



## *Significance testing of parameters*

- › The SIENA software estimates parameters  $\beta_k$  and their standard errors  $\text{st.err.}(\beta_k)$ .
- › By calculating the ***t-ratio*** of those, parameter significance can be tested:

$$t = \beta_k / \text{st.err.}(\beta_k)$$

- is approximately normally distributed\*
- under the assumption (null hypothesis) that *actual network evolution* follows a model in which the parameter is constrained to zero ( $\mathbf{H}_0: \beta_k = \mathbf{0}$ ).

\* Thus far, this claim largely rests on extensive simulation studies.



## **D: Model specification and parameter interpretation for network evolution models**

- › When investigating social network dynamics, researchers ususally do not come empty-handed but have theories or hypotheses about the mechanisms that might operate.
- › These mechanisms [hopefully] can be expressed in terms of SIENA parameters, and the theories and hypotheses can be restated in terms of the corresponding model parameters.
- › By estimating the parameters and calculating significance tests for them, the theories / hypotheses can be tested empirically.



## *Example (Torlò, Steglich, Lomi & Snijders, 2007)*

- › **75 students** enrolled in an MBA program;
- › **4 network variables:** advice-seeking, communication, friendship, acknowledge-contribution-to-learning;
- › **co-evolving behavioural dimension:** performance in examinations;
- › **several other actor variables:** gender, age, experience, nationality;
- › **3 waves** in yearly intervals.

We focus here on the analysis of the evolution of the advice network only.

What theories / hypotheses are investigated? *[just 3 of them...]*



## 1. “You seek advice from your friends.”

Mechanism: presence of a friendship tie between two actors increases the likelihood that an advice tie is present between the same actors.

If  $x_{ij}$  stands for  $i$  seeking advice from  $j$  and  $w_{ij}$  stands for  $i$  naming  $j$  as a friend, then the effect

$$s_{i \text{ friend}}(x) = \sum_j x_{ij} w_{ij}$$

operationalises the above mechanism, and the corresponding parameter  $\beta_{\text{friend}}$  can be used to test it.



- › The effect statistic  $\mathbf{s}_{i \text{ friend}}$  counts the degree to which advice seeking and friendship ‘overlap’.
- › The parameter  $\beta_{\text{friend}}$  expresses whether by changing the advice network, such an overlap is sought or avoided, i.e., whether friendship enhances or weakens advice seeking:

$\beta_{\text{friend}}$  positive: advice seeking is more likely when it coincides with friendship;

$\beta_{\text{friend}}$  negative: advice seeking is less likely when it coincides with friendship.

- › In SIENA, the effect can be included as main effect of a dyadic covariate (friendship) on network evolution.



## 2. “The lower your performance, the more advice you need [and the more you will seek].”

Mechanism: actors with low performance scores are likely to have more outgoing advice ties than actors with high performance scores.

If  $z_i$  stands for performance of actor  $i$ , then the effect

$$s_{i \text{ own-performance}}(x) = z_i \sum_j x_{ij}$$

operationalises the above mechanism, and the parameter  $\beta_{\text{own-performance}}$  can be used to test it.



- › The effect statistic  $s_{i \text{ own-performance}}$  counts the degree to which active advice seeking and performance coincide.
- › The parameter  $\beta_{\text{own-performance}}$  expresses whether by changing the advice network, such an coincidence is sought or avoided, i.e., whether own performance enhances or weakens advice seeking:

$\beta_{\text{own-performance}}$  positive: high performers seek more advice than low performers;

$\beta_{\text{own-performance}}$  negative: high performers seek less advice than low performers.

- › In SIENA, the effect can be included as an ego-effect of an actor variable (performance) on network evolution.



3. “The higher your performance, the better the advice you can give [and the more you will be asked for advice].”

Mechanism: actors with high performance scores are likely to attract more incoming advice ties than actors with low performance scores.

Let  $z_j$  now stand for performance of actor  $j$ , then effect

$$s_{i \text{ others-performance}}(x) = \sum_j z_j x_{ij}$$

operationalises the above mechanism, and the parameter  $\beta_{\text{others-performance}}$  can be used to test it.



- › The effect statistic  $s_{i \text{ others-performance}}$  counts the degree to which passive advice seeking ('being asked') and performance coincide.
- › The parameter  $\beta_{\text{others-performance}}$  expresses whether by changing the advice network, such a coincidence is sought or avoided, i.e., whether others' performance makes them more or less attractive as sources of advice:

$\beta_{\text{others-performance}}$  positive: high performers are more often asked for advice than low performers;

$\beta_{\text{own-performance}}$  negative: high performers are less often asked for advice than low performers.

- › In SIENA, the effect can be included as an alter-effect of an actor variable (performance) on network evolution.

	parameters	Advice seeking
Network evolution components	<i>network rate period 1</i>	9.24***
	<i>network rate period 2</i>	7.13***
	<i>outdegree</i>	-3.36**
	<i>reciprocity</i>	0.54***
	<i>transitive triplets</i>	0.27***
	<i>friendship</i>	0.35***
	<i>communicatio</i>	1.11***
	<i>gender similarity</i>	0.17*
	<i>gender ego</i>	-0.19*
	<i>gender alter</i>	-
	<i>GPA alter</i>	-
	<i>age similarity</i>	-
	<i>age ego</i>	-
	<i>age alter</i>	-
	<i>experience alter</i>	0.18*
	<i>nationality similarity</i>	0.40***
	<i>nationality alter</i>	-
	<i>performance similarity</i>	-
	<i>performance ego</i>	-0.11***
<i>performance alter</i>	0.15***	

## Results on these particular hypotheses:

(excerpts from Torlò et al.'s Table 5)

Significantly positive parameter  $\beta_{\text{friend}}$

**Advice is sought from friends.**

Significantly negative parameter

$\beta_{\text{own-performance}}$

**Low performers seek more advice.**

Significantly positive parameter

$\beta_{\text{others-performance}}$

**High performers are more likely asked for advice.**



- › *The SIENA manual* contains formulae for all effects that currently can be estimated.
- › Behavioural dimensions that co-evolve with the network are modelled analogously.
- › Thus far, the method does not [yet] allow to draw conclusions about the macro level.

Simulation studies can be used to do so:

- first estimate a reasonable model,
- use estimates to generate many “similar” data sets,
- investigate distribution of macro properties over simulated [& observed] data,
- draw conclusions about how micro mechanisms lead to [or mediate] macro outcomes.



## More on interpretation of parameter estimates

Several types of interpretation:

1. At face value: parameter values and odds
2. Preference? Constraint? Artifact?
3. In relation to the data
4. As extrapolation into the distant future?

Beware: **model-based inference!**



## ***1. Parameter values...***

The linear shape of the objective function allows to compare effects of different predictor variables directly.

- *Parameters for two effects with same scale (e.g., “same gender” and “same ethnicity”) can be directly compared,*
- *otherwise, scaling needs to be taken into account (e.g., “reciprocity” and “transitive triplets”)*

***Note that such comparisons take place on the objective function’s scale – NOT on some tangible outcome measure!***

***[A predicament common to all logistic models.]***



## **... and odds**

The local characterisation of the model allows to calculate conditional odds and conditional odds ratios.

- *The impact of a unit difference in statistic  $s_{ik}$  on the odds of choosing  $x^a$  vs.  $x^b$  is given by  $\exp(\beta_k)$ .*
- *Odds ratios  $\exp(\beta_k)/\exp(\beta_m) = \exp(\beta_k - \beta_m)$  allow to compare different effects' sizes.*
- *From both, binary (or other) comparison probabilities can be calculated.*

**Note that while such comparisons take place on the probability scale, they refer to rather artificial choice situations!**



## 2. Preference? Constraint? Artifact?

Typically, the parameter estimated for the outdegree statistic  $S_{i \text{ outdg.}} = \sum_j X_{ij}$  is quite significantly negative.

*Does this mean social actors prefer not to have social ties?*

- Suppose  $\beta_k = -2.6$ .
- Then the odds of having another tie vs. not having it are  $\exp(\beta_k) = \exp(-2.6) = 0.07$
- And the binary probability to have one vs. not to have one is  $\exp(\beta_k) / (1 + \exp(\beta_k)) = 0.07 / 1.07 = 0.07 = 7\%$
- ***This reflects the overall density of the network!***



## ***Zero objective function = density 50%***

An objective function that does not discriminate between options implies model actors' indifference to everything – hence, all ties will be present (or absent) with equal probability. The density then will be 50%.

*Because most networks commonly studied have a density way below 50% (and hence also most network evolution processes take place in a low-density region of the network space), the outdegree parameter is estimated as significantly negative.*

***Similar arguments can be made about other parameters, BUT beware of control effects in the model!***



## ***Zero reciprocity effect = “reciprocity index equals density”***

An objective function that *controls for density* but does not discriminate between reciprocation of existing ties and creation of asymmetric ties has a reciprocity effect of zero. The probability of a reciprocated tie then is identical to the probability of any tie, which is the density.

*Because many networks commonly studied have a reciprocity index way above the density (and hence also most network evolution takes place in such network regions), the reciprocity parameter is estimated as significantly positive.*

***The more effects are controlled for, the more difficult it gets to tie parameters to descriptive measures...***



## ***Beware of data collection artifacts!***

As shown above, the outdegree parameter typically is estimated as significantly negative, reflecting a lower than 50% density of the network.

*In many data collection designs, it is impossible to ever obtain a density of 50% (e.g., “Pick up to 12 best school friends, from your cohort of size 100+”).*

*Hence, the parameter’s significant departure from zero must not be treated as “result” of an analysis!*

*Its inclusion in a model must be viewed as the necessary control for density, without which other conclusions cannot be obtained.*



## ***Don't mis-diagnose constraint as preference!***

Several parameters may not necessarily reflect the expression of actual preference in the actors' decisions, but features of the opportunity structure they face when making them.

### ***Case in point: transitive closure.***

*"Friends of my friends are my friends"*

*... because I prefer to attain cognitive balance?*

*... or because I have a higher chance to interact with them?*

***Unique conclusion typically not possible without validation by additional data.***

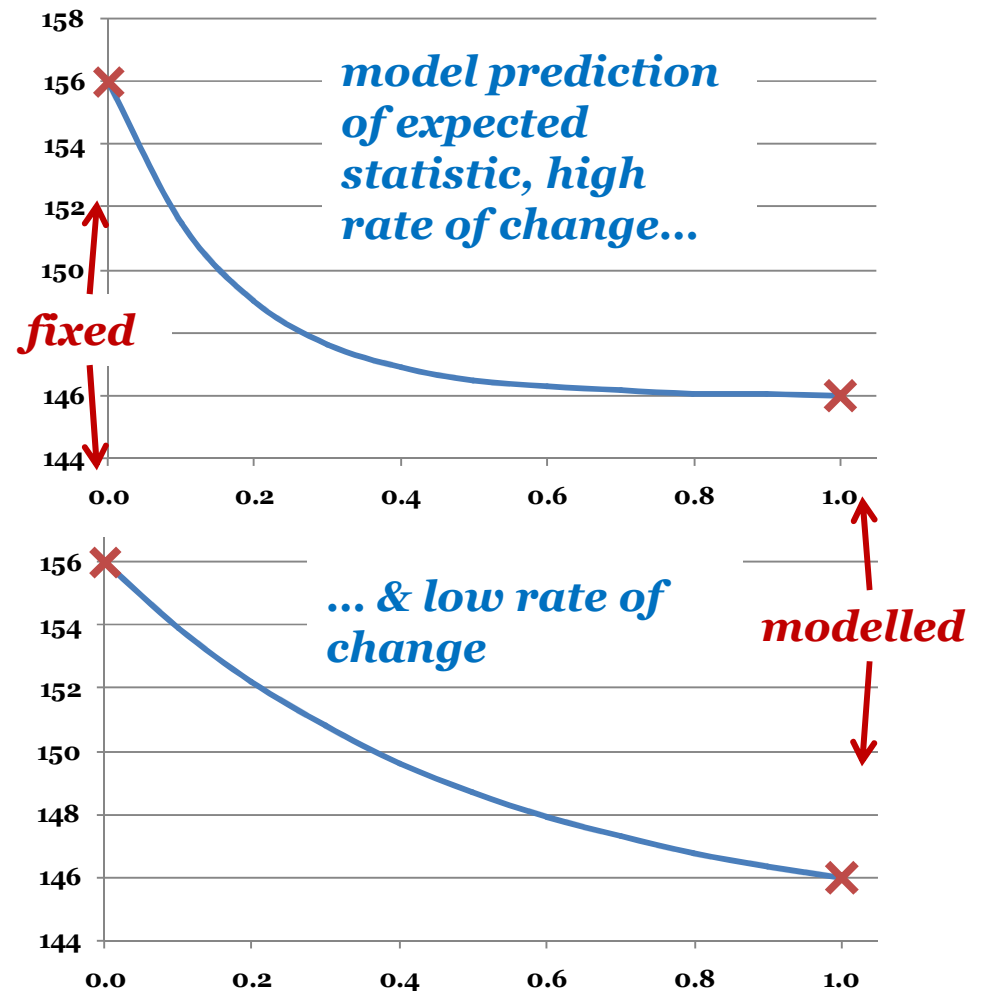


### 3. Interpolated region

Suppose a modelled network statistic  $s$  changes from **156** to **146** during an observation period.

The corresponding parameter is adjusted such that data point **146** is “hit” in expected value when starting out from **156**.

Steepness of the curve is co-determined by the total amount of change in the period (as modelled by rate parameters).





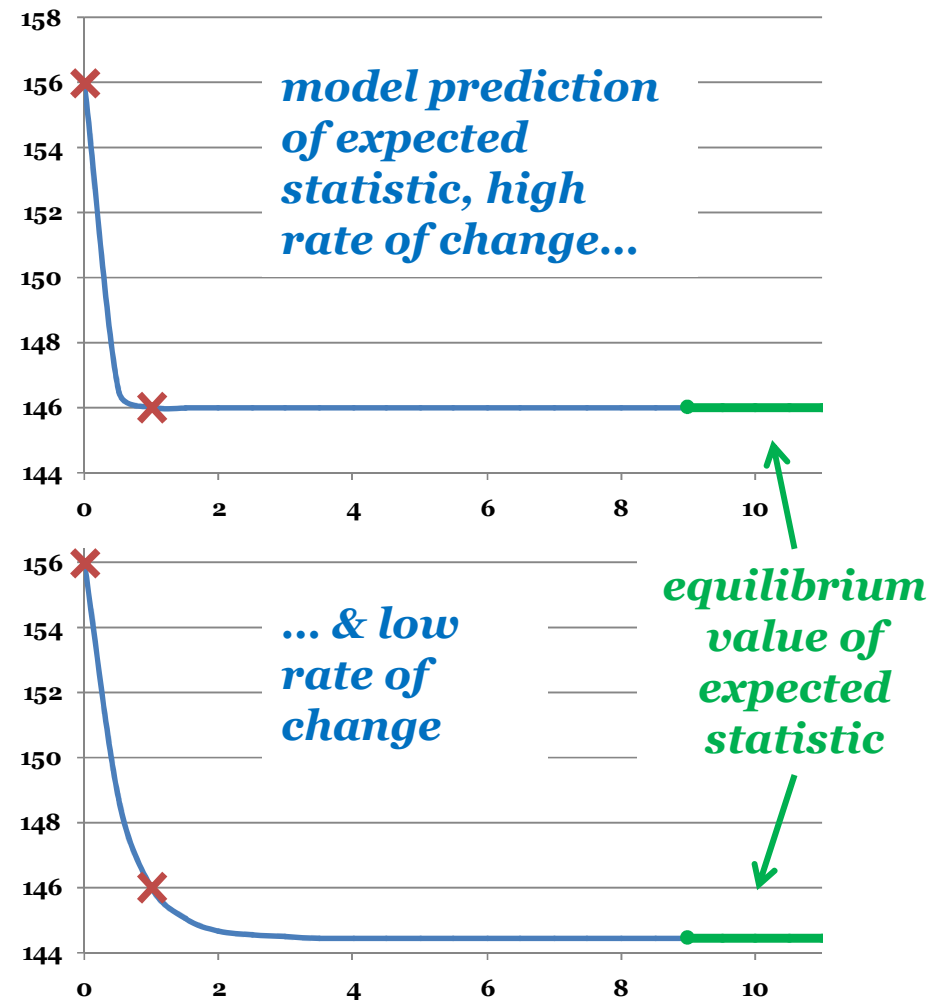
## 4. Projected equilibrium

Like all Markov processes, these models eventually lock in to an **equilibrium distribution** (here: on the network space).

This equilibrium...

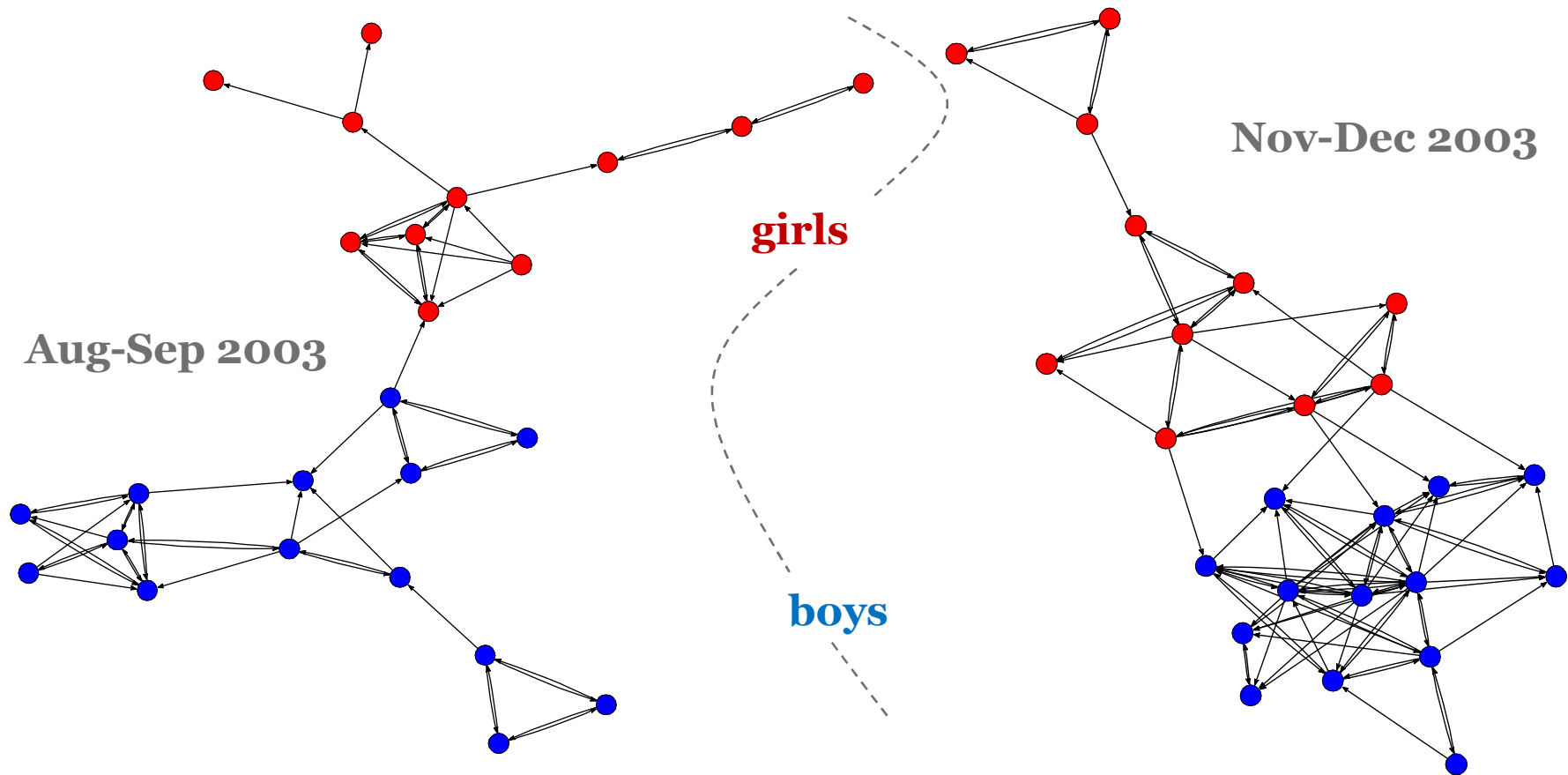
- is uniquely identified by the parameter estimates,
- hence does not allow to draw conclusions about the observation period!

But it allows...anything??





## *Now consider this classroom friendship network:*





## *Analyse this network during class...*

- › ... making use of the following effects:
  - outdegree (density),
  - reciprocity,
  - transitive triplets,
  - gender effects of sender and receiver,
  - a gender homophily effect.
- › Formulate expectations (hypotheses) relating to these effects,
- › test the hypotheses based on SIENA output.